MAT 91112 Opgave E32

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Funktionen f har Taylorpolynomiet (udviklingspunkt 0)

$$P_4(x) = 1 + \frac{1}{6}x^2 + \frac{1}{120}x^4$$

Da $f^{(k)}(0) = P_4^{(k)}(0)$ for $k = 0 \ldots 4$ fås

$$f(0) = 1, f'(0) = 0, f''(0) = \frac{1}{3}, f'''(0) = 0, f^{(4)}(0) = \frac{1}{5}$$

Da $f^{(5)}(0) = 0$ har vi $P_5(x) = P_4(x)$ for alle $x \in R$. Hermed finder vi for $|x| \leq \frac{1}{2}$ og idet vi udnytter, at vi da har $\left|f^{(6)}(x)\right| \leq \frac{1}{6}$

$$\begin{aligned}
|f(x) - P_4(x)| &= |f(x) - P_5(x)| = \left|\frac{1}{6!}f^{(6)}(\xi)\,x^6\right| \\
&\leq \frac{1}{6!} \cdot \frac{1}{6} \cdot \left(\frac{1}{2}\right)^6 = \frac{1}{720} \cdot \frac{1}{6} \cdot \frac{1}{64} \\
&= \frac{1}{276\,480} < \frac{1}{250000} = 4 \cdot 10^{-6}
\end{aligned}$$