MAT 91112 Opgave E18

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Vi skal bestemme grænseværdien

$$\lim_{x\to\infty}\left(\sqrt{16x^4+x^2+5}-\sqrt{16x^4-3x^2-7}\right)$$

Vi finder med $A=16x^4+x^2+5$ og $B=16x^4-3x^2-7$ at

$$\begin{aligned}
&\sqrt{16x^4+x^2+5}-\sqrt{16x^4-3x^2-7}\\
=\;&\frac{\left(\sqrt{A}-\sqrt{B}\right)\left(\sqrt{A}+\sqrt{B}\right)}{\sqrt{A}+\sqrt{B}}=\frac{A-B}{\sqrt{A}+\sqrt{B}}=\frac{4x^2+12}{\sqrt{A}+\sqrt{B}}\\
=\;&\frac{4x^2+12}{x^2\left(\sqrt{16+x^{-2}+5x^{-4}}+\sqrt{16-3x^{-2}-7x^{-4}}\right)}\\
=\;&\frac{4+12x^{-2}}{\sqrt{16+x^{-2}+5x^{-4}}+\sqrt{16-3x^{-2}-7x^{-4}}}\to\frac{4}{\sqrt{16}+\sqrt{16}}=\frac{1}{2}
\end{aligned}$$

for $x\to\infty$.