MAT 91121-22 Opgave E34

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1. Kurveintegralet
$$\int_k xy^2 dx + \left(x^2 y + y\right) dy = 0$$
da differentialformen $xy^2 dx + \left(x^2 y + y\right) dy$ er eksakt og da kurven k er lukket. At differentialformen er eksakt følger af, at
$$\frac{\partial}{\partial y}\left(xy^2\right) = 2xy = \frac{\partial}{\partial x}\left(x^2 y + y\right)$$
overalt i R^2.

2. Det andet kurveintegral udregnes ved brug af Greens sætning:
$$\begin{aligned}
\int_k xy^2 dx + x^3 dy &= \iint_S \left(-2xy + 3x^2\right) dA \\
&= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} d\theta \int_0^1 \left(-2r^2 \cos\theta \sin\theta + 3r^2 \cos^2\theta\right) r dr \\
&= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} d\theta \int_0^1 \left(-2\cos\theta \sin\theta + 3\cos^2\theta\right) r^3 dr \\
&= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{4}\left(-2\cos\theta \sin\theta + 3\cos^2\theta\right) d\theta \\
&= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{2}\sin\theta d \sin\theta + \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \frac{3}{4}\frac{1+\cos(2\theta)}{2} d\theta \\
&= \left[-\frac{1}{4}\sin^2\theta\right]_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} + \frac{3}{8}\left[\theta + \frac{\sin(2\theta)}{2}\right]_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \\
&= 0 + \frac{3}{8}\left(\frac{3\pi}{2} - 1\right) = \frac{9}{16}\pi - \frac{3}{8}
\end{aligned}$$