

MAT 91122 Opgave E38

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Vi skal finde det 2. Taylorpolynomium for

$$f(x, y) = 3 \cos(x - y) + \ln(x^2 y^3)$$

med udviklingspunkt $(1, 1)$.

Den generelle formel for P_2 er

$$\begin{aligned} P_2(x, y) &= f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b) \\ &\quad + \frac{1}{2} \left(f_{xx}(a, b)(x - a)^2 + 2f_{xy}(a, b)(x - a)(y - b) + f_{yy}(a, b)(y - b)^2 \right) \end{aligned}$$

Vi finder, idet vi udnytter, at $\ln(x^2 y^3) = 2 \ln x + 3 \ln y$,

$$\begin{aligned} f_x(x, y) &= -3 \sin(x - y) + \frac{2}{x} \\ f_y(x, y) &= 3 \sin(x - y) + \frac{3}{y} \\ f_{xx}(x, y) &= -3 \cos(x - y) - \frac{2}{x^2} \\ f_{xy}(x, y) &= 3 \cos(x - y) \\ f_{yy}(x, y) &= -3 \cos(x - y) - \frac{3}{y^2} \end{aligned}$$

Hermed har vi, at $f(1, 1) = 3$, $f_x(1, 1) = 2$, $f_y(1, 1) = 3$, $f_{xx}(1, 1) = -5$, $f_{xy}(1, 1) = 3$, $f_{yy}(1, 1) = -6$. Altså fås

$$P_2(x, y) = 3 + 2(x - 1) + 3(y - 1) - \frac{5}{2}(x - 1)^2 + 3(x - 1)(y - 1) - 3(y - 1)^2$$