MAT 91122 Opgave E41

Preben Alsholm
IFAK, DTU

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Massebalanceligningerne

$$\begin{aligned}\frac{d}{dt}\left(2y\left(t\right)\right) &= e^{-t}-2y\left(t\right)+u\left(t-2\right)z\left(t-2\right)\\ \frac{d}{dt}\left(3z\left(t\right)\right) &= 2te^{-t}-4z\left(t\right)+2y\left(t\right)+3\delta\left(t-3\right)\end{aligned}$$

Ved Laplacetransformation fås

$$\begin{aligned}2s\overline{y}\left(s\right)-2y\left(0\right) &= \frac{1}{s+1}-2\overline{y}\left(s\right)+e^{-2s}\overline{z}\left(s\right)\\ 3s\overline{z}\left(s\right)-3z\left(0\right) &= \frac{2}{\left(s+1\right)^2}-4\overline{z}\left(s\right)+2\overline{y}\left(s\right)+3e^{-3s}\end{aligned}$$

Idet vi udnytter, at $y\left(0\right)=0=z\left(0\right)$, fås

$$\begin{aligned}2\left(s+1\right)\overline{y}\left(s\right)-e^{-2s}\overline{z}\left(s\right) &= \frac{1}{s+1}\\ -2\overline{y}\left(s\right)+\left(3s+4\right)\overline{z}\left(s\right) &= \frac{2}{\left(s+1\right)^2}+3e^{-3s}\end{aligned}$$

Løsning af disse to ligninger med de to ubekendte $\overline{y}\left(s\right)$ og $\overline{z}\left(s\right)$ kan f.eks. foretages ved brug af determinantmetoden (Cramers regel):

$$\begin{aligned}\overline{z}\left(s\right) &= \frac{\begin{vmatrix}2\left(s+1\right) & \frac{1}{s+1}\\ -2 & \frac{2}{\left(s+1\right)^2}+3e^{-3s}\end{vmatrix}}{\begin{vmatrix}2\left(s+1\right) & -e^{-2s}\\ -2 & 3s+4\end{vmatrix}}\\ &= \frac{6\frac{e^{-3s}\left(s+1\right)^2+1}{s+1}}{2\left(3s+4\right)\left(s+1\right)-2e^{-2s}}=\frac{3e^{-3s}\left(s+1\right)^2+3}{\left(3s+4\right)\left(s+1\right)^2-\left(s+1\right)e^{-2s}}\end{aligned}$$